2.1.23

Suppose $\mathbf{x}$ is any solution to $A\mathbf{x} = \mathbf{0}$. Then $CA\mathbf{x} = I_n\mathbf{x} = \mathbf{x}$. However, $C\mathbf{0} = \mathbf{0}$ therefore $\mathbf{x} = \mathbf{0}$.

If A were to have more columns than rows, there would have to be at least one free variable in the row-reduced echelon form of A and hence there would be a non-trivial solution to the homogeneous equation $A\mathbf{x} = \mathbf{0}$. As we have shown, A has <u>no</u> non-trivial solutions to the homogeneous equation and hence cannot have more columns than rows.

2.3.11

a) **True**. By Theorem 8(b,d).
b) **True**. By Theorem 8(e,h).
c) **False**. For a counterexample take $A = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.
d) **True**. By the contrapositive statement to Theorem 8(c,d).
e) **True**. If A is invertible then so is A^T with $(A^T)^{-1} = (A^{-1})^T$.

2.9.3

We need to solve the system of equations

$$\begin{bmatrix} 1 & -2 & -3 \\ -4 & 7 & 7 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & -3 \\ 0 & -1 & -5 \end{bmatrix} \qquad (R_2 \to R_2 + 4R_1)$$
$$\sim \begin{bmatrix} 1 & -2 & -3 \\ 0 & 1 & 5 \end{bmatrix} \qquad (R_2 \to -R_2)$$
$$\sim \begin{bmatrix} 1 & 0 & 7 \\ 0 & 1 & 5 \end{bmatrix} \qquad (R_1 \to R_1 + 2R_2)$$

So

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} 7 \\ 5 \end{bmatrix}$$